SOLUTIONS

Joint Entrance Exam | IITJEE-2019

9th APRIL 2019 | Morning Session

Joint Entrance Exam | JEE Mains 2019

PART-A	PHYSICS

1.(3) Given $m = 5 \Rightarrow \frac{-v}{u} = 5 \Rightarrow v = -5u$

$\therefore \frac{1}{-5u} + \frac{1}{u} = -\frac{1}{40} \Rightarrow u = 32 \text{ cm}$

2.(3) $B_{net} = \sqrt{B_0^2 + B_1^2} = 10^{-6} \times \sqrt{904}$

$\therefore \quad E = C \times B_{net} = 3 \times 10^8 \times 10^{-6} \times \sqrt{904}$

$\therefore \quad F = QE = 10^{-4} \times 3 \times 10^8 \times 10^{-6} \times \sqrt{904} = 0.9 \text{ N.}$

$\therefore \quad F_{rms} = \frac{F}{\sqrt{2}} = 0.6\text{N}$

3.(1) $R_{eq} = 6 + \frac{12 \times 4}{12 + 4} = 9\Omega \quad \therefore i_{main} = \frac{72}{9} = 8 \text{ amp.}$

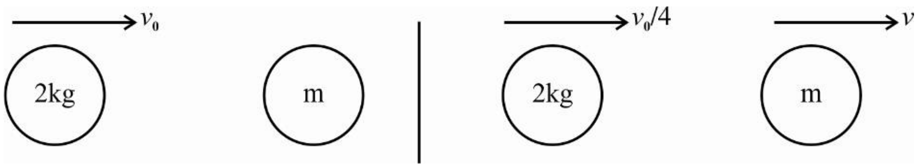
$\therefore \quad i_{10\Omega} = \frac{4}{12+4} \times 8 = 2 \text{ amp.}$

$\therefore \quad Q = \text{CV} = 10 \times 10^{-6} \times (10 \times 2) = 200\ \mu C$

4.(3) Clearly $\frac{2\pi}{\lambda} = 0.157 \quad \therefore \lambda = 40\text{m}$

Hence for 4th harmonic, $l = 4 \times \frac{\lambda}{2} = 80\,\text{m}$

5.(4)



COM gives : $2v_0 = 2 \times \frac{v_0}{4} + \text{mv}$ ...(1)

For elastic collision $v - \frac{v_0}{4} = v_0$ ...(2)

From (1) and (2) $m = 1.2$ kg

6.(3) $U_{system} = \frac{1}{4\pi\varepsilon_0}\left[\frac{(Q \times q)}{D + d/2} + \frac{Q(-q)}{(D - d/2)} + \frac{2 \times (-q)}{d}\right]$

$= \frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d} - \frac{2qd}{D^2}\right] \quad \left(\because D^2 - \frac{d^2}{4} = D^2\right)$

7.(3) We have $V_{r.m.s.} \propto \sqrt{T}$

$\therefore \frac{200}{V} = \sqrt{\frac{273 + 127}{273 + 227}} \Rightarrow V = 100\sqrt{5} \text{ m/s}$

8.(3) Conservation of energy gives $\frac{1}{2} I_{IAOR} \omega^2 = \text{mgh}$

For ring $\frac{1}{2}\times 2mR^2\times\left(\frac{V}{R}\right)^2=\text{mgh}_1$ $\Rightarrow h_1=\frac{v^2}{g}$

For Cylinder, $\frac{1}{2}\times 3\text{m}\left(\frac{\text{R}}{2}\right)^2\left(\frac{\text{v}}{\text{R}/2}\right)^2=\text{mgh}_2$ $\Rightarrow \text{h}_2=\frac{3}{2}\frac{\text{v}^2}{\text{g}}$

For sphere $\frac{1}{2}\cdot\frac{7}{5}m\left(\frac{R}{4}\right)^2\times\left(\frac{V}{R/4}\right)^2=\text{mgh}_3$ $\therefore\ h_3=\frac{7}{10}\frac{v^2}{g}$

$\therefore\ h_1:h_2:h_3=1:\frac{3}{2}:\frac{7}{10}=10:15:7$

9.(3) Net repulsive force

$F=F_1-F_2=\frac{\mu_0}{2\pi}\frac{i_1i_2\times a}{a}-\frac{\mu_0}{2\pi}\frac{i_1i_2}{2a}\times a$

$=\frac{\mu_0}{4\pi}i_1i_2$

10.(1) Theoretical

11.(3) For no drifting, $\sin\theta=\frac{u}{v_m}=\frac{2}{4}=\frac{1}{2}$

$\therefore\ \theta=30°$

Hence direction with flow of rivers is

$\alpha=180-\theta=120°$

12.(1) Work-done will be equal to rise in potential energy of suspended section

$\therefore\ W=\frac{mg}{L}\times\frac{L}{n}\times\frac{L}{2n}=\frac{MgL}{2n^2}$

13.(4) H-Cl is diatomic molecule hence degree of freedom is

$f=3$ (translational) + 2 (rotational) + 1 (vibration) = 6

$\therefore\ \frac{1}{2}m\overline{v}^2=\frac{6}{2}K_BT\quad\therefore T=\frac{m\overline{v}^2}{6K_B}.$

14.(3) $V^2=u^2-2gh\ \Rightarrow\ m^2v^2=m^2u^2-2\text{mgh}$

$\Rightarrow\ p^2=A-Bh$ (where $A=m^2u^2,\ b=2\text{mg}$)

Clearly it is parabolic functions.

Starting from ground momentum first decreases and then becomes zero at highest position. Therefore it increases in –ve direction. Hence 3rd option is correct.

15.(4) $V=i_g[R_g+R]$

$=4\times10^{-3}[50+5000]=20.2$ volt

≈ 20 volt.

16.(2) $W=|\Delta U|=\frac{Q^2}{2C_1}-\frac{Q^2}{2C_2}=3.75\times10^{-6}\text{J}$

17.(4) At 0°C, $v=\frac{\omega}{K}=\frac{1000}{3}\text{m/s}$ $\therefore$ $\frac{1000/3}{336}=\sqrt{\frac{273}{273+T}}$ $(\because V\propto\sqrt{T})$ $\therefore$ $T\approx 277\ K=4°\text{C}$

18.(4) $\frac{1}{2}I\omega^2 = K\theta^2 \Rightarrow \frac{1}{2}I \times 2\cancel{\omega} \times \frac{d\omega}{dt} = K \times 2\theta \cdot \frac{d\cancel{\theta}}{\cancel{dt}}$ $\quad\therefore\quad \alpha = \frac{d\omega}{dt} = \frac{2k\theta}{I}$

19.(3) $R_{AB} = R_{BC} = R_{CD} = \frac{R}{4}$ & $R_{DE} = R_{EC} = \frac{R}{8}$ $\quad\therefore\ R_{eq} = \frac{\left(\frac{3R}{4}+\frac{R}{8}\right)\times\frac{R}{8}}{R} = \frac{7}{64}R$

20.(1) $\frac{\lambda_2}{660} = \frac{\frac{1}{2^2}-\frac{1}{3^2}}{\frac{1}{2^2}-\frac{1}{4^2}} \Rightarrow \lambda_2 = 488.88 \approx 488.9\text{Å}$

21.(3) If θ_c is contact angle, then for equilibrium of rises water in tube, we have $T\cos\theta \times 2\pi r = \text{Mg}$
Hence if r is doubled, rises mass will also doubled.

22.(3) $E_p = \frac{GM}{(3a)^2} + \frac{G.2M}{(3a)^2} = \frac{GM}{3a^2}$

23.(1) $\tau = \mu B\sin\theta = (niA)B\sin\theta = 100\times 3\times\frac{5\times 2.5}{100\times 100}\times 1\times\frac{1}{\sqrt{2}} = 0.265\text{N} \approx 0.27\text{N}$

24.(1) According to question shift = width of n fringe pattern
$\Rightarrow (\mu-1)t = n\times\frac{D}{a}\cdot\lambda \quad\therefore\ t = \frac{nD\lambda}{a[\mu-1]}$

25.(2) $T = 2\pi\sqrt{\frac{l}{g}}$ & $T' = 2\pi\sqrt{\frac{l}{g-(g/16)}}$ $\quad$ ($\because$ Buoyancy $= \frac{mg}{16}$)

$\therefore \quad T' = T\times\frac{1}{\sqrt{1-\frac{1}{16}}} = 4T\times\sqrt{\frac{1}{15}}$

26.(4) For a solenoid, self inductance is given as
$L_{\text{self}} = N\times A\times\mu_0\left(\frac{N}{L}\right) \quad\therefore\ L_{\text{self}} \propto \frac{1}{L}$

27.(3) input resistance = $r_1 = \frac{10\text{mV}}{15\times 10^{-6}} \approx 0.67\times 10^3\,\Omega = 6.0\times 10^2\,\Omega = 0.67\text{ K}\Omega$

Voltage gain = $+\text{B}\frac{\text{Rc}}{\text{r}} = +\frac{3\text{mA}}{15\mu\text{A}}\times\frac{1\text{K}\Omega}{0.67\text{K}\Omega} = +300$ Volt

28.(3) Clearly $\frac{2\pi}{\lambda} = \frac{2\pi}{5\times 10^{-7}}$ $\quad\therefore\ \lambda = 5000\text{Å}$

$\therefore\ E = eV + \phi \quad\Rightarrow\ V = \frac{12375}{5000} - 2 = 0.475\text{ V} \approx 0.48$ volt .

29.(1) Area under curve in process `A' on P-V diagram is more. Hence
$W_A > W_B \ \therefore\ \Delta Q_A > \Delta Q_B$ & $\Delta U_A = \Delta U_B$

30.(4) $\rho = \frac{10}{(0.1)^3} = 10^{+4}\text{ kg/m}^3$

Also $\rho = \frac{M}{l^3} \quad\therefore\ \frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} + \frac{3\Delta l}{l} = \frac{0.1}{10} + \frac{0.01}{0.1}\times 3 \qquad = \frac{1}{100} + \frac{3}{10} = \frac{31}{100} = 0.31\text{kg/m}^3$

PART-B	CHEMISTRY

1.(2) from given eq:

$$\begin{array}{ll} N_2(g) + 3H_2(g) \longrightarrow 2NH_3(g) & \\ 1\text{mole} \quad\; 3\text{mole} & \\ 1\times 28\text{gm} \;\; 3\times 2\text{gm} & \\ = 28\text{gm} \quad = 6\text{gm} & \end{array}$$

for option -1 $\because$ 6 gm H_2 reacts $\bar{c}$ 28 gm N_2 i.e. both reactants utilized completely.

for option -2 $\because$ 6 gm H_2 reacts $\bar{c}$ 28 gm N_2

$\therefore$ 10 gm H_2 reacts $\frac{28}{6}\times 10\,\text{gm}\ N_2$

$= 46.67\,\text{gm}\ N_2$ required, however amount. of N_2 given = 56 gm

i.e. N_2 present in excess.

So, H_2 utilizes completely & hence limiting reagent.

for option-3 $\because$ 6 gm H_2 reacts $\bar{c}$ 28 gm N_2

$\therefore$ 4 gm H_2 reacts $\bar{c}$ $\frac{28\times 4}{6}\text{gm}\,N_2$

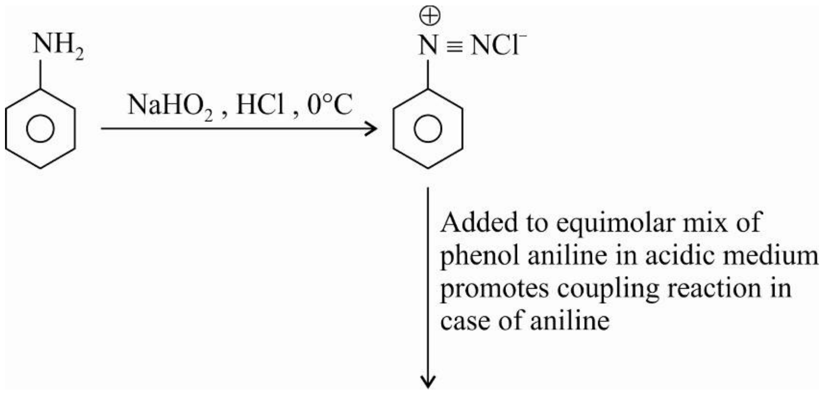
$= 18.67\text{gm}\,N_2$ required. But N_2 given = 14 gm.

$\therefore N_2$ is L. R.

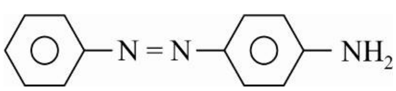
for option - 4 $\because$ 6gm H_2 reacts $\bar{c}$ 28 gm N_2

$\therefore$ 6 gm H_2 reacts $\bar{c}$ $\frac{28\times 8}{6}\text{gm}\ N_2$ $= 37.33\,\text{gm}\,N_2$ required but N_2 given = 35 gm.

2.(2)



$\therefore$ major product should be



3.(3) *q* & *w* are path functions.

Option – 3 is correct

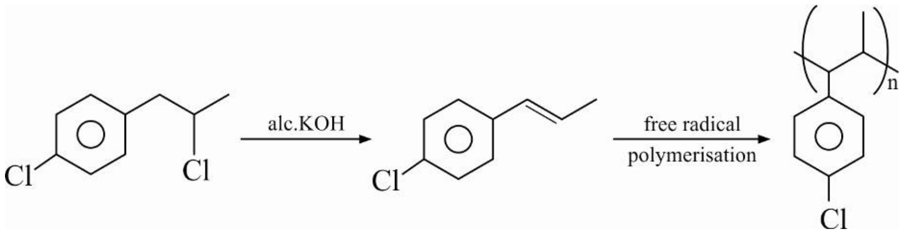
4.(4) Aerosol is a colloidal solution of solid in air. Eg. Smoke, dust etc. are considered as aerosol.

Option – 4 is correct

5.(1) $Mg \xrightarrow{air, \Delta} MgO + Mg_3N_2$

Option – 1 is correct

6.(2)



7.(4) from Raoult's law: from Dalton's law:

$P_M = x_M \cdot P_M^0$ $\quad$ $P_M = Y_M \cdot P_T$

& $P_N = x_N \cdot P_N^0$ $\quad$ $P_N = Y_N \cdot P_T$

$\therefore \quad x_M \cdot P_M^0 = Y_M \cdot P_T$ ...... (i)

& $\quad x_N \cdot P_N^0 = Y_N \cdot P_T$ ...... (ii)

eq. (i) ÷ eq. (ii)

$$\frac{x_M \cdot P_M^0}{x_N \cdot P_N^0} = \frac{Y_M \cdot P_T}{Y_N \cdot P_T}$$

$$\frac{x_M}{x_N} \times \frac{450}{700} = \frac{Y_M}{Y_N}$$

$$\frac{x_M}{x_N} = \frac{14}{9} \times \frac{Y_M}{Y_N}$$

$$\therefore \quad \boxed{\frac{x_M}{x_N} > \frac{Y_M}{Y_N}}$$

8.(1) $CuSO_4 \cdot 5H_2O$

$[Cu(H_2O)_4]SO_4 \cdot H_2O$

One H_2O molecule is present as water of crystallization.

9.(4) for EMR $C = \nu\lambda \quad \therefore \nu = \frac{C}{\lambda}$

From Planck's formula,

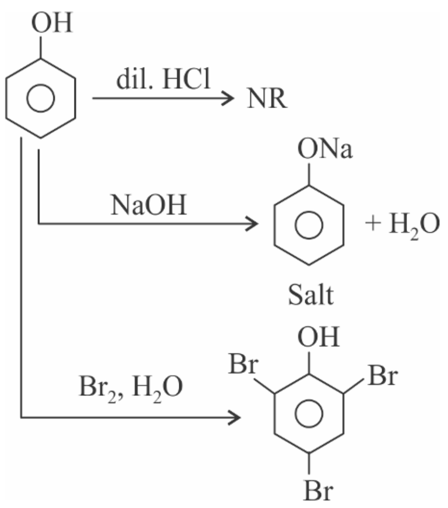
$$E = h\nu \qquad \therefore \nu = \frac{E}{h} \qquad \therefore \frac{C}{\lambda} = \frac{E}{h}$$

So, $$\frac{\Delta\nu_{Lyman}}{\Delta\nu_{Balmer}} = \frac{(\nu_{max} - \nu_{min})_{Lyman}}{(\nu_{max} - \nu_{min})_{Balmer}} = \frac{(E_{max} - E_{min})_{Lyman}}{(E_{max} - E_{min})_{Balmer}}$$

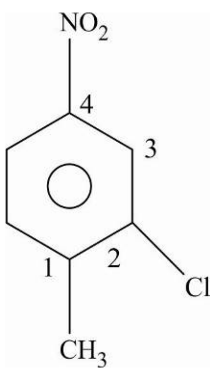
$$= \frac{13.6\left(\frac{1}{1} - \frac{1}{\alpha}\right) - 13.6\left(\frac{1}{1} - \frac{1}{4}\right)}{13.6\left(\frac{1}{4} - \frac{1}{\alpha}\right) - 13.6\left(\frac{1}{4} - \frac{1}{9}\right)} \qquad = \frac{1 - 0 - 1 + \frac{1}{4}}{\frac{1}{4} - 0 - \frac{1}{4} + \frac{1}{9}} = \frac{\frac{1}{4}}{\frac{1}{9}} = \frac{9}{4}$$

$\therefore\ 9 : 4$

10.(4)



11.(4)



2-chloro-1-methyl-4-nitrobenzene

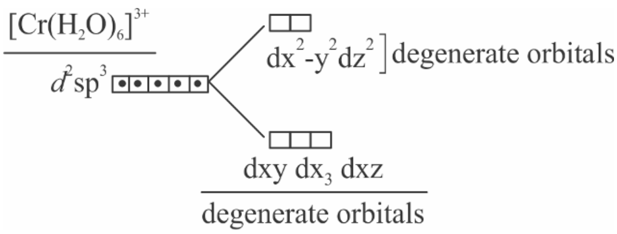
12.(1) Increase in CO_2 induces Global warming.

13.(4) Sucrose is formed due to bonding between.

α – Glucose from C_1 position and β – fructose from C_2 position.

14.(4) $CH_3-CH=CH-\overset{\overset{\displaystyle O}{\|}}{C}-O-CH_3 \xrightarrow{LiAlH_4} CH_3-CH=CH-CH_2-OH$

15.(4)



16.(2)

$$CH_3-C\equiv CH \xrightarrow{DCl} CH_3-\overset{\overset{\displaystyle Cl}{|}}{C}=CHD$$

$$\downarrow DI$$

$$CH_3-\underset{\underset{\displaystyle I}{|}}{\overset{\overset{\displaystyle Cl}{|}}{C}}-CHD_2$$

17.(4) $K = 1s^2 2s^2 2p^6 3s^1$

I.P.$_{1st}$ <<< I.P. $_{2nd}$.

$$\frac{K^+ = 1s^2 2s^2 2p^6}{\text{inert gas configuration}}$$

18.(3) $\xrightarrow[\text{Activating nature}]{-CN\ \ -Cl\ \ -Me\ \ -OMe}$

i.e. $D < A < C < B$

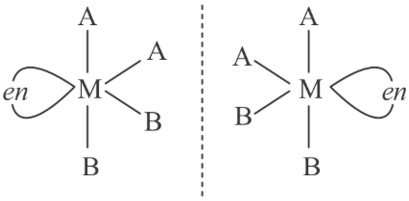
19.(4) Cryolite = $Na_3[AlF_6]$

20.(2) $Cu^{2+} + Zn \longrightarrow Zn^{2+} + Cu$

$n = 2$

$\Delta G^\circ = -nfE^\circ_{cell} = -2 \times 96500 \times 2 = -386$ KJ

21.(1)



$(M(AA)A_2B_2)type$

No symmetry

22.(3) C_{60} contains 12 Pentagons of 20 hexagons

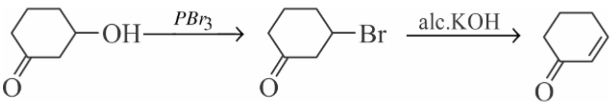
23.(3) V_2O_5 ——— H_2SO_4

$TiCl_4 / Al(Me)_3$ ——— Polyethylene

$PdCl_2$ ——— Ethanal

Iron oxide ——— NH_3

24.(4)



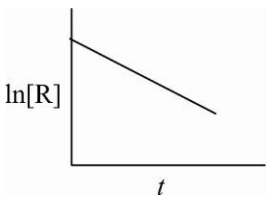
25.(1)

$R \longrightarrow P$

$a \qquad x$

$a - x \qquad x$

For 1st order reaction



$+\frac{dx}{dt} = k(a-x)^1$

$\int \frac{dx}{a-x} = \int kdt$

$-\ln(a-x) = kt + c$

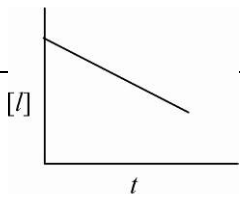
If $t = 0$

$c = -\ln a \qquad \therefore \qquad -\ln(a-x) = kt - \ln a$

or $\underset{y}{\ln(a-x)} = \underset{m\,x}{-kt} + \underset{C}{\ln a}$

Similarly

$\underset{(A)}{\underset{(A_0)}{R}} \longrightarrow P$



$$\frac{-d[R]^{(A)}}{dt} = k[R]^0$$

or $\quad -d(R) = kdt$

or $\quad \int d[R] = -\int kdt$

$[R] = -kt + c$

or $\quad [A] = -kt + c$

$y = mx + c$

26.(3) For xy

$i = 2$ as complete dissociation takes place.

$\pi_{xy} = 2 \times CRT$

For $BaCl_2$

$i = 3$

$\pi_{BaCl_2} = 3 \times 0.01 \times RT = 0.03\ RT$

As $\pi_{xy} = 4 \times \pi_{BaCl_2}$

$$\frac{\pi_{xy}}{\pi_{BaCl_2}} = \frac{2 \times C \times RT}{0.03\,RT}$$

$$\frac{4 \times \pi_{BaCl_2}}{\pi_{BaCl_2}} = \frac{2 \times C}{0.03}$$

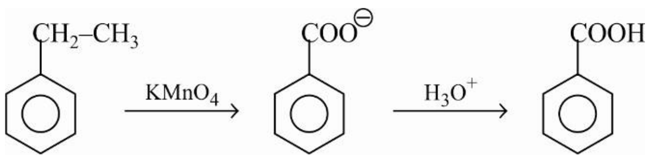
$$C = \frac{4 \times 0.03}{2 \times 100} = 6 \times 10^{-2}$$

27.(3) $T_C = \frac{8a}{27Rb}$

For Kr the value of $\left(\frac{a}{b}\right)$ is highest

Thus T_C is also highest

28.



29.(3) $\overset{+2}{N}O$, $\overset{+1}{N_2}O$, $\overset{+4}{N}O_2$, $\overset{+3}{N_2}O_3$

30.(1) For C_2, $Z_2 = 12$

$BO = 2.5$

$\therefore$ For C_2^-, $Z_2 = 13$

$BO_{C_2^-} = 2.5$ (filling in BMO)

$BO_{O_2^-} = 2.5$ (filling in ABMO)

$BO_{NO^-} = 2.0$

$BO_{F_2^-} = 0.5$

PART-C	MATHEMATICS

1.(1) The points $(1,\ f(1)) = (1,-2)$ and $(-1,\ f(-1)) = (-1,0)$

So, slope of line joining the two pts $= \dfrac{-2}{2} = -1$

Again, slope of tangent to $y = x^3 - x^2 - 2x$ is $\dfrac{dy}{dx} = 3x^2 - 2x - 2$

$\Rightarrow \quad 3x^2 - 2x - 2 = -1 \quad \Rightarrow \quad 3x^2 - 2x - 1 = 0 \quad \Rightarrow \quad x = \dfrac{2 \pm \sqrt{4+12}}{6} = -\dfrac{1}{3}, 1$

2.(2) Since S.D. $= \sqrt{\dfrac{\sum xi^2}{x} - \left(\dfrac{\sum xi}{x}\right)^2} = \sqrt{\dfrac{(-1)^2 + 0^2 + (1)^2 + K^2}{4} - \left(\dfrac{(-1)+0+(1)+k}{4}\right)^2}$

$= \sqrt{\dfrac{K^2+2}{4} - \dfrac{K^2}{16}} \qquad \Rightarrow \ 3K^2 + 8 = 16 \times 5 = 80$

$= \sqrt{\dfrac{3K^2+8}{4}} \qquad \Rightarrow \ K^2 = \dfrac{72}{3} = 24$

Now, $\dfrac{\sqrt{3K^2+8}}{4} = \sqrt{5} \qquad \Rightarrow \ K = 2\sqrt{6}$

3.(3) $\displaystyle\int_0^2 \frac{\sin^3 x}{\sin x + cosx}\, dx = I\,(say) \quad \Rightarrow \quad I = \int_0^{\pi/2} \frac{\cos^3 x}{\sin x + \cos x} dx$

$\Rightarrow \quad 2I = \displaystyle\int_0^{\pi/2} \frac{\sin^3 x + \cos^3 x}{\sin x + cosx} dx \quad \Rightarrow \quad 2I = \int_0^{\pi/2} (\sin^2 x + \cos^2 x - \sin x \cos x) dx$

$\Rightarrow \quad 2I = \displaystyle\int_0^{\pi/2} \left(1 - \frac{1}{2}\sin 2x\right) dx \quad \Rightarrow \quad 2I = x + \frac{\cos 2x}{4}\Bigg]_0^{\pi/2}$

$\Rightarrow \quad 2I = \left(\dfrac{\pi}{2} - \dfrac{1}{4}\right) - \left(\dfrac{1}{4}\right) = \dfrac{\pi}{2} - \dfrac{1}{2} \quad \Rightarrow \quad I = \dfrac{\pi - 1}{4}$

4.(1) $\cos^2 10° - \cos 10° \cos 50° + \cos^2 50°$

$= \dfrac{1}{2}\left[2\cos^2 10° - 2\cos 10° \cos 50° + 2\cos^2 50°\right] = \dfrac{1}{2}\left[1 + \cos 20° - \cos 40° - \cos 60° + 1 + \cos 100°\right]$

$= \dfrac{1}{2}\left[\dfrac{3}{2} + \left(\cos 20° - \cos 40°\right) + \cos\left(90° + 10°\right)\right] = \dfrac{1}{2}\left[\dfrac{3}{2} + 2 \cdot \sin 30° \sin 10° - \sin 10°\right] = \dfrac{3}{4}$

5.(2) $2\cos^2\theta + 3\sin\theta = 0 \quad \Rightarrow \quad 2 - 2\sin^2\theta + 3\sin\theta = 0$

$\Rightarrow \quad 2\sin^2\theta + 3\sin\theta - 2 = 0 \quad \Rightarrow \quad 2\sin^2\theta - 4\sin^2\theta + \sin\theta - 2 = 0$

$(2\sin\theta + 1)(\sin\theta - 2) = 0$

$\Rightarrow \quad \sin\theta = -1/2 \quad \Rightarrow \quad \theta = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{-5\pi}{6}, \frac{-\pi}{6} \Rightarrow \quad \frac{7\pi + 11\pi - 5\pi - \pi}{6} = \frac{12\pi}{6} = 2\pi$

6.(1) $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{4} = \lambda \quad \Rightarrow \quad$ Point on the line $= (2\lambda + 1, 3\lambda - 1, 4\lambda + 2)$

Now for p

$(2\lambda + 1) + 2(3\lambda - 1) + 3(4\lambda + 2) = 15 \quad \Rightarrow \quad 20\lambda + 5 = 15 \quad \Rightarrow \quad \lambda = \frac{1}{2}$

$\Rightarrow \quad p = \left(2, \frac{1}{2}, 4\right) \quad \Rightarrow \quad OP = \sqrt{4 + \frac{1}{4} + 16} = \sqrt{20 + \frac{1}{4}} = \frac{9}{2}$

7.(2) $f(x) = \frac{x^2}{1-x^2} = \frac{1}{1-x^2} - 1 \quad \Rightarrow \quad A = R \setminus [-1, 0) \quad \Rightarrow \quad f(x) \in (-\infty, -1) \cup [0, \infty)$

8.(3) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}\ldots\ldots\begin{bmatrix} 1 & n-1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 78 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \quad \Sigma(n-1) = 78 = \frac{n(n-1)}{2} = 78 \quad \Rightarrow \quad n(n-1) = 156 = 12 \times 13 \quad \Rightarrow \quad n = 13$

$\Rightarrow \quad \begin{bmatrix} 1 & 13 \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -13 \\ 0 & 1 \end{bmatrix}$

9.(3) $f'(x) = \lambda x(x-1)(x+1) \Rightarrow \quad f'(x) = \lambda x^3 - \lambda x \quad \Rightarrow \quad f(x) = \frac{\lambda x^4}{4} - \frac{\lambda x^2}{2} + \mu$

For $f(x) = f(0)$

$\Rightarrow \quad \frac{\lambda x^4}{4} - \frac{\lambda x^2}{2} = 0 \Rightarrow \quad \frac{\lambda x^2}{4}(x^2 - 2) = 0 \quad \Rightarrow \quad x = 0, \sqrt{2}, -\sqrt{2}$

10.(1) $\frac{M(24+18)}{\sqrt{24 - 18M^2}} = 7\sqrt{3} \Rightarrow 6M = \sqrt{3}\sqrt{24 - 8M^2}$

$\Rightarrow \quad 36M^2 = 72 - 54M^2 \quad \Rightarrow \quad 90M^2 = 72 \Rightarrow M \pm \frac{2}{\sqrt{5}}$

11.(2) Let equation of the plane be $a(x-0) + b(y+1) + c(z-0) = 0$

$\Rightarrow \quad ax + by + cz = -b \quad \ldots\ldots(1)$

Now $a(0-0)+b(0+1)+c(1-0)=0$

$\Rightarrow \quad b+c=0 \quad \ldots\ldots(2)$

and $\dfrac{a\times 0+b\times 1+c\times(-1)}{\sqrt{a^2+b^2+c^2}\ \sqrt{2}}=\cos\dfrac{\pi}{4}=\dfrac{1}{\sqrt{2}}$

$\Rightarrow \quad b-c=\sqrt{a^2+2b^2}$

$2b=\sqrt{a^2+2b^2}$

$4b^2=a^2+2b^2$

$\Rightarrow \quad a^2=2b^2 \quad \Rightarrow \quad a=\pm\sqrt{2}b$

So, equation of plane $\pm\sqrt{2}x+y-z=-1$

$(\sqrt{2},+4)$ satisfy.

12. $\vec{\beta}_1=\lambda\vec{\alpha}=3\lambda\hat{i}+\lambda\hat{j} \quad \Rightarrow \quad 2\hat{i}-\hat{j}+3\hat{k}=(3\lambda-\mu)\hat{i}+(\lambda+3x)\hat{j}+\delta k.$

$\vec{\beta}_2=\mu\hat{i}-3\mu\hat{j}+\mu\hat{k}$

Also, $\vec{\beta}=\vec{\beta}_1-\vec{\beta}_2 \quad \Rightarrow \quad 3\lambda-\mu=2$ and $\delta=-3$

$\lambda+3\mu=-1$

So, $\lambda=1/2$ and $\mu=-1/2$

$\Rightarrow \quad \vec{\beta}_1=\dfrac{1}{2}\left(3\hat{i}+\hat{j}\right)$ and $\vec{\beta}_2=-\dfrac{1}{2}\hat{i}+\dfrac{3}{2}\hat{j}+3\hat{k}$

$$\vec{\beta}_1\times\vec{\beta}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{3}{2} & 1/2 & 0\\ -\frac{1}{2} & \frac{3}{2} & -3\end{vmatrix}=\hat{i}\left(-\frac{3}{2}\right)-\hat{j}\left(-\frac{9}{2}\right)+\hat{k}\left(\frac{9}{4}+\frac{1}{4}\right)=\frac{1}{2}\left(-3\hat{i}+9\hat{j}+3\hat{k}\right)$$

13.(3) $Sn=50n+\dfrac{n(n-7)}{2}A \quad \Rightarrow \quad T_n=S_n-S_{n-1}$

$\Rightarrow \quad T_n=50\,(n-n+1)+\dfrac{A}{2}\left[n(n-7)-(n-1)\,(n-8)\right]$

$T_n=50+\dfrac{A}{2}\left[-7n+9n-8\right]$

$\Rightarrow \quad T_n=50+A(n-4)$ and $d=T_5-T_4=(50+A)-(50)=A$

$\Rightarrow \quad a_{50}=50+46A$

$(d, 9_{50}) = (A, 50 + 46A)$

14.(4) $k = \lim\limits_{x \to \frac{\pi}{4}} \dfrac{\sqrt{2}\cos x - 1}{\cot x - 1} = \lim\limits_{x \to \frac{\pi}{4}} \dfrac{1 - \sqrt{2}\cos x}{\text{cosec}^2 x} = \lim\limits_{x \to \frac{\pi}{4}} \sqrt{2}\sin^3 x = \sqrt{2} \times \left(\dfrac{1}{\sqrt{2}}\right)^3 = \dfrac{1}{2}$

15.(2) $z = \dfrac{\alpha + i}{\alpha - i} \quad \Rightarrow \quad |z| = \dfrac{|\alpha + i|}{|\alpha + i|} \quad \Rightarrow \quad |z| = 1$

So, circle with radius 1.

16.(3) $T_4 = {}^6C_3 \left(x^{\log_8 x}\right)^3 \left(\dfrac{2}{x}\right)^3 = 20 \times 8^7 \qquad \Rightarrow \qquad x^{\log_8 x} = 64x$

$\Rightarrow \qquad 20 \times 8 \times \dfrac{\left(x^{\log_8 x}\right)^3}{x^3} = 20 \times 9^7 \qquad \Rightarrow \qquad (\log 8x)^2 = \log_0 64 + \log_8 x$

$\Rightarrow \qquad \dfrac{x^{\log_8 x}}{x} = 64 \qquad \Rightarrow \qquad \left(\log_8 x - 2\right)\left(\log_8 x + 1\right) = 0 \Rightarrow x = 64, \ \dfrac{1}{8}$

17.(2) $g(x) = f(f(x)) = 15 - |f(x) - 10|$

$= 15 - |15 - |x - 10| - 10|$

$\Rightarrow \qquad g(x) = 15 - |5 - |x - 10|| \qquad \Rightarrow \qquad g(x) = 15 - |15 - x|;\ x \geq 10$

$15 - |x - 5| \ \ x < 10 \qquad \Rightarrow \qquad g(x) = 10 + x;\ \ x < 5$

$20 - x;\ 5 \leq x < 10$

$x;\ 10 \leq x < 15$

$30 - x; \quad 15 \leq x$

So, $g(x)$ is not differentiable at $x = 5, 10, 15$

18.() $p, q \in R$ The other root is $2 + \sqrt{3}$

$p.q \in R \Rightarrow$ The other root is $2 + \sqrt{3}$

So, $p = -4 \ \ q = 1$

$p^2 - 4q = 16 - 4 = 12$

$p^2 - 4q - 12 = 0$

19.(4) $I = \displaystyle\int \sec^{2/3} x \cos ecx^{4/3} x\, dx = \int \frac{dx}{\sin^{4/5} x \cos^{2/5} x} = \int \frac{\sec^2 x\, dx}{(\tan x)^{4/3}}$

$\Rightarrow \ I = \int \frac{d(\tan x)}{\tan^{4/3}} = \frac{(\tan x)^{1-\frac{4}{3}}}{1-\frac{4}{3}} + C = -3(\tan x)^{-1/3} + C$

20.(4) $y = x^3 + ax - b \ \Rightarrow \ \frac{dy}{dx} = 3x^2 + a$

Now $\left.\frac{dy}{dx}\right|_{(1,-5)} = -1$

$\Rightarrow \quad a + 3 = -1 \quad \Rightarrow \quad a = -4$

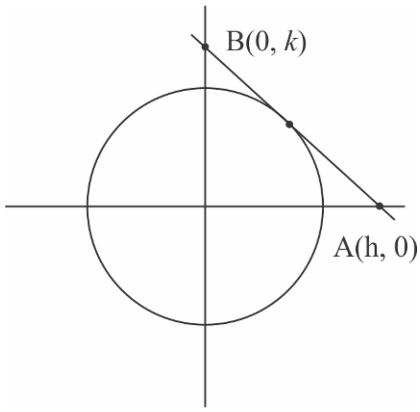
$(1,-5)$ lies on the curve

$\Rightarrow \quad -5 = 1 - 4 - b$

$b = 5 - 3 = 2$

So, curve is $y = x^3 - 4x - 2$

21.(4) $\Rightarrow$ Mid-point $A, B = P = \left(\frac{h}{2}, \frac{k}{2}\right)$



$\Rightarrow \quad h = 2x, k = 2y$

Now line as: $\frac{x}{h} + \frac{y}{x} = 1$ is layout

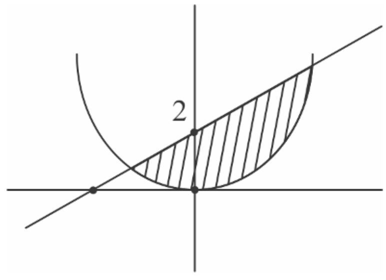
$\Rightarrow \quad \frac{\left|\frac{0}{h} + \frac{0}{k} - 1\right|}{\sqrt{\frac{1}{h^2} + \frac{1}{k^2}}} = 1 \quad \Rightarrow \quad \frac{1}{h^2} + \frac{1}{x^2} = 1$

So, locus: $\frac{1}{4x^2} + \frac{1}{4y^2} = 1 \quad \Rightarrow \quad x^2 + y^2 = 4x^2y^2 \ \Rightarrow \ x^2 + y^2 - 4x^2y^2 = 0$

22.(3) Path $= 1 - \left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{3}\right) = 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{7}{8} = 1 - \frac{7}{32} = \frac{25}{32}$

23.(2) $A = \left\{(\lambda, 4); x^2 \le y \le x + 2\right\}$

$y \ge x^2$ and $y - x - 2 \le 0$



Point of intersection

$\Rightarrow \quad x^2 = x + 2 \ \Rightarrow \ x^2 - x - 2 = 0$

$x = -1, 2$

$$A=\left[\int_{-1}^{2}\left((x+2)-x^2\right)dx=\frac{x^2}{2}+2x-\frac{x^3}{3}\right]_{-1}^{2}=\left(2+4-\frac{8}{3}\right)-\left(\frac{1}{2}-2+\frac{1}{3}\right)$$

$$=8-3-\frac{1}{2}=5-\frac{1}{2}=\frac{9}{2}$$

24.(2) $f(x+y)=f(x)\cdot f(y)\ \Rightarrow\ f(x)=a^x$

$f(1)=2\ \Rightarrow\ f(x)=2^x$

$$\sum_{k=1}^{10}f(a+k)=2^a\left[2+2^2+...+2^{10}\right]=\frac{2^{a+1}\left(z^{10}-1\right)}{2-1}$$

Also, $2^{a+1}\left(2^{10}-1\right)=16\left(2^{10}-1\right)$

$\Rightarrow\quad a+1=4\ \Rightarrow\ a=3$

25.(2) Roots of $x^2+x+1=0$ are ω,ω^2

$\Rightarrow\quad \alpha=\omega,\beta=\omega^2\ \Rightarrow\ 1+\alpha+\beta=0$

$$\begin{vmatrix}y+1&\alpha&\beta\\\alpha&y+\beta&1\\\beta&1&y+\alpha\end{vmatrix}=\begin{vmatrix}y&\alpha&\beta\\y&y+\beta&1\\y&1&y+\alpha\end{vmatrix}=y\begin{vmatrix}1&\alpha&\beta\\0&y+\alpha&1-\beta\\0&1-\alpha&y+\alpha-\beta\end{vmatrix}$$

$(C_1\to C_1+C_2+C_3)$ $\qquad (R_2\to R_2-R_1)\ \ (R_3\to R_3-R_1)$

$$=y\begin{vmatrix}1&\alpha&\beta\\0&(y-i\sqrt{3})&1-\beta\\0&1-\alpha&(y+i\sqrt{3})\end{vmatrix}$$

$$=y\{(y^2+3)-(1-\alpha-\beta+\alpha\beta)\}$$
$$=y\{y^2+3-(2+1)\}=y^3$$

26.(3) $x\dfrac{dy}{dx}+2y=x^2\ \Rightarrow\ \dfrac{dy}{dx}+\left(\dfrac{2}{x}\right)y=x$

I.F. $=e^{\int\frac{2}{x}dx}=x^2$

So, $yx^2=\int x^3dx\ \Rightarrow\ yx^2=\dfrac{x^4}{4}+c$

$y(1)=1\ \Rightarrow\ 1=\dfrac{1}{4}+C\Rightarrow C=\dfrac{3}{4}$

So, set $\ y=\dfrac{x^2}{4}+\dfrac{3}{4x^2}$

27.(2) $y^2 = 16x$ if point is (1, 4) $\Rightarrow t_1 = \frac{1}{2} = (at_1^2, 2t_1)$

Again, for focal chord $t_1 t_2 = -1 \Rightarrow \quad t_2 = -2$

So, length of focal chord = $a(t_1 - t_2) = 4\left(\frac{1}{2} + 2\right)^2 = 25$

28.(4) Let the equation of line be $x = 2 + r\cos\theta, y = 3 + r\sin\theta$

For, $r = 24$

$(2 + 4\cos\theta) + (3 + 4\sin\theta) = 7$

$\Rightarrow \quad 4(\cos\theta + \sin\theta) = 2 \quad \Rightarrow \quad 4(\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta) = 1$

$\Rightarrow \quad 4\tan^2\theta + 8\tan\theta + 4 = \sec^2\theta \quad \Rightarrow \quad 3\tan 2\theta + 8\tan\theta + 3 = 0$

$\Rightarrow \quad \tan\theta = \frac{-8 \pm \sqrt{64-36}}{6}$

$\Rightarrow \quad \tan\theta = \frac{-8 \pm 2\sqrt{7}}{6} \quad \Rightarrow \quad tan\,\theta = \frac{2\sqrt{7}-8}{6}$

$$\tan\theta = \frac{2\sqrt{7}-8}{6} = \frac{-\left(\sqrt{7}-1\right)^2}{(7-1)} = \frac{-\sqrt{7}-1}{\sqrt{7}+1}$$

$$\tan\theta = \frac{1-\sqrt{7}}{1+\sqrt{7}}$$

29.(3) $pV(\sim p \wedge q) \equiv (pV \sim p) \wedge (pVq) \equiv pVq$

So, $\sim (pVq) \equiv (\sim p) \wedge (\sim q)]$

30.(3) $M = {}^8C_6 \times {}^5C_5 + {}^8C_7 \times {}^5C_4 + {}^8C_8 \times {}^5C_3 = n \quad \Rightarrow \quad M = n = 78$